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ABSTRACT

This document consists of two major parts. The first, titled Initial Comments, discusses the motivation for creating this material, provides a list of minimal characteristics in calculators for various educational levels, notes a number of basic assumptions on calculator use, and discusses curriculum concerns regarding effects of calculators. The second part of this document details suggested uses of calculators for (1) kindergarten through grade 3; (2) grades 4 through 8; and (3) grades 9 through 12. The material concludes with a bibliography of references on calculators in pre-college education. (MP)

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USES OF THE CALCULATOR
IN SCHOOL MATHEMATICS, K-12

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USES OF THE CALCULATOR IN SCHOOL MATHEMATICS, K-12

Part I Initial Comments

Introduction

The motivation for this MCTM Monograph on handhald calculators came from a meeting of leaders in mathematics education at Ann Arbor in February, 1975. The personnel gathered at that meeting recommended that the Michigan Council of Teachers of Mathematics sponsor such a publication. Sally Ross, Sam Kaner, Richard Kahle, Robert Peterson and Arthur Coxford agreed to accept the responsibility for writing the monograph. Before the first meeting of the committee Alfred Capoferi joined the work force.

In keeping with the MCTM Monograph tradition, the committee decided to emphasize the practical uses of the calculator in grades K-12. In order to ensure that the materials and suggestions would be of maximal use to classroom teachers, materials were prepared individually. Then the committee came together to critically evaluate each other's work.

The committee also decided that it would not engage in a discussion of whether or not the calculator should be used in the classroom. Many scholars throughout the nation are presently collecting empirical evidence on the positive and negative effects of its use.(1) This evidence should help educators make rational decisions in the future.

The electronic calculator is here and readily available to students. Thus the committee's position was to present suggestions for classroom use that were in accord with approved classroom methods and learning objectives. The electronic calculator is recognized as another teaching tool available to the teacher--much like hundreds charts, flannel board and trigonometric tables--but one whose uses have not been thoroughly investigated. In the spirit of making helpful suggestions for use of a new tool, we give you this monograph.

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(1) Suydam, Marilyn. Electronic Hand Calculators: The Implications for Pre-College Education. Washington, D.C., NSF (Grant no. EPP 75-16157), 1976.

Calculators

The market place is now replete with electronic calculators with many different characteristics. For classroom use, especially in the elementary and middle schools, the following minimal characteristics seemed desirable to the committee.

A. A display of at least eight digits
B. A display that is easy to read by two or more people
C. Easily accessible keys which are easily operated
D. Four basic functions - addition, subtraction, multiplication, and division
E. Floating decimal; decimal is placed in answer automatically
F. Clearing keys for display and for entry
G. Algebraic logic - enter statement in machine
H. No dual purpose keys
I. AC adapter

If the machines are to be used in group settings, it is desirable to have all of the machines the same. This will allow instruction to proceed more smoothly without special attention to different procedures on a variety of machines. In any case both the teacher and the students should become familiar with their machines before attempting group projects.

At higher levels in school, the necessity for similar machines decreases. Additional machine capabilities are greatly desired at the high school level. The following features should be considered when purchasing classroom sets of machines.

J. Change sign key
K. Constant key
L. Scientific notation
M. Simple memory or parentheses
N. Square root function
O. Reciprocal function
P. Logarithmic function
Q. Trigonometric functions

Basic Assumptions on Calculator Use.

A number of assumptions were made regarding the use of calculators of any kind. These guided the committee in its task.

1. The use of calculators should not replace instruction in skills and concepts.

   When teaching concepts and basic skills, the calculator is a useful tool. For example, it may be used to verify results, to illustrate concepts, or to release students from tedious calculation while trying to learn a new algorithm such as division. It may be used to focus on one aspect of an algorithm such as a partial product in multiplication. It may also be used to solve an equation after the equation has been converted into a form for which the calculator is useful. However, it will not replace instruction in the multiplication algorithm nor replace instruction in solving equations.

2. The calculator can be a useful teaching-learning device in a mathematics classroom.

3. In courses where computational skill is not a major objective, the calculator may be used to decrease the student's computational load.

   In courses such as consumer mathematics, trigonometry, analysis, calculus, and probability and statistics the calculator should be used. Such use will allow more realistic problems to be considered, will free the student from small computational errors, will require the student to use fundamental ideas to write "calculator programs," will replace looking up values in tables, and will allow concentration on the ideas and problems to be solved.

4. Use of the calculator should be taught to all students before they complete their formal mathematics instruction.

   The calculator has provided the schools with a solution to a previously unsolvable problem--what to do for the learner who has never been able to execute a certain algorithm regardless of the extent of instruction. Now it is possible to show how to arrive at an answer on the machine. Fundamental skill in computation is necessary for all persons. If it cannot be attained without the assistance of the electronic calculator, it should be attained with it.

Curriculum Concerns

Possible effects of extensive use of the calculator on the curriculum cannot be detailed with certainty. Suydam's[2] work mentioned earlier discusses the possibilities fully. However, several issues

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[2] Suydam; op. cit.

are raised immediately which, once identified, should be considered by each teacher at the level affected. The first concerns the introduction of decimal notation. The calculator reports in decimals, and they arise as soon as division is introduced. Should decimal work be introduced earlier to children?

A related issue is the role of common fractions. Since most calculators function in decimal notation, should common fractions be downplayed? Also, what initial notation for fractions should be used--decimal or common? If decimals are introduced earlier, many aspects of the curriculum will also be affected, so an extensive curriculum study is needed.

Another issue is related to negative numbers. If a student keys in 5 - 9 = he gets -4. This could be done as early as the kindergarten or first grade by students. Does this suggest that there is a need to consider the earlier introduction of negative number concepts?

If mastery of some algorithmic work is assumed to be still desirable, the question arises as to what level of skill is appropriate. Should a student be able to multiply two two-digit numbers or a three-digit by a two-digit number? Similar questions arise with respect to division and addition of whole numbers and decimals. Some decisions are needed if we are to use the calculator and not put the student at a disadvantage. School districts need to develop guidelines for calculator use so that students who transfer are not at a disadvantage.

The use of calculators requires additional work in estimation. The students need to be able to develop "ball park" answers so that they do not accept a wrong answer displayed by the calculator. Approximate calculation also calls for rounding skills--skills that are not now emphasized in our schools.

Certainly these issues are but a few of those that may arise with extensive use of electronic calculators. Teachers need to be aware of some of these questions, so that when a student asks "What is -5?" or "How come $1 \div 7$ is so big?" they can respond rationally.

PART II Suggested Uses of Calculator

Part II is divided into major sections: Grades K-3, Grades 4-8, and Grades 9-12. In the first two sections the topic is identified by an underlined heading at the left of the page. This is followed by a brief statement of an objective related to calculator use. Finally, the underlined words "How To" appear. These words identify the beginning of the suggestions for use of the calculator in attaining the objective.

The third and last section of Part II, "Grades 9-12", is organized in a similar manner, but with some modification. On the left-hand side of the page the topic or topics are identified by name. Illustrations of activities are found immediately to the right of the topic heading. These illustrations tend to emphasize how to set something up for effective calculator use. The needed mathematical knowledge is assumed.

This collection is not intended to, nor be, exhaustive. It is not intended that a teacher should use all the suggestions given for a particular level. Each teacher should choose those activities which are appropriate.

GRADES K-3

Topic

Counting and Numeral Recognition: To count sets of objects, one to nine, and display the numeral on the calculator.

How to: Teachers display sets of objects. Students count the objects and show the number telling how many on the calculator. Students can compare calculator displays or teacher can examine to check accuracy. Practice in reading numerals can be included by asking students to say the number name shown.

Number Recognition: Given a set of number cards, one through five, and the selection of one of the cards, the learner produces the number.

How to: Display number cards one through five. Ask a student to point to a card. The students with calculators must show that number on the calculator. This can be extended by asking a student to name the number. A good beginning exercise with calculators for kindergarten and first grade students.

Number Production: Given an oral name, a written name or a set of objects, the learner produces the correct symbol on the calculator.

How to: This objective can be approached in several ways for a variety of experiences with number recognition and production.

1. The teacher writes a number on the board and the student enters the same number on the calculator.
2. The teacher orally names a number and the student pushes the correct button.
3. The teacher shows the student concrete objects or draws pictures of some objects on the board. The student writes the number on a piece of paper and then pushes that numbered button. The teacher checks that the number is correct both on the paper and on the calculator.

Counting on the Calculator: Given student mastery of the initial idea of counting from 1-10, the student displays the sequence of individual numerals 1-10.

How to: Ask the class to count slowly from 1 to 10. Record the sequence for reference. Have students key in the following sequence:

1 ⊕ 1 ⊕ 1 ⊕ 1 ...
step 1 step 2 step 3 step 4

Compare the results after each step with the reference sequence to see that it produces the numbers 1-10 in sequence. Use this procedure to count by 1's on the calculator. Extend when appropriate to counting by 2's, 3's, 10's, etc. by keying ⊕2 or +③ etc. in sequence. Prior to this activity, show students how to produce 1-more, 2-more...than a given number on the calculator.

Rote Counting: To count to twenty and check on the calculator.

How to: After the student says each number he checks the symbol by "counting on the calculator." (See previous section).

Rational Counting: Shown a picture of a set of from one to ten objects, the learner will count while pointing to appropriate objects.

How to: Student marks off each object in a set with a pencil mark as he pushes the "+" and "1" button.

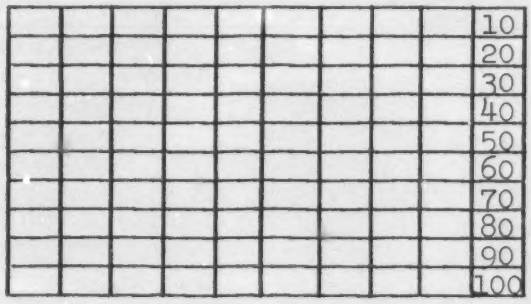
Counting by Tens: After the teacher says 10, 20, 30...100 in sequential order, the student will write the given numerals.

How to: Students can write the given numeral and then check answers on the calculator as they "count by 10's" on the calculator.

Counting On: Given orally any of the following numbers, 10, 20, 30...90 the student will say the counting sequence of the following nine numbers. Example 30, 31, 32,...

K-3 (cont'd)

How to: This is a good beginning for using the calculator to discover patterns. One student may write the starting number and then write each succeeding number as it comes up on the display on a one inch (or 2 centimeters) grid paper as shown below.



Several students could choose different beginning points (10: student a), (20: student b), (30: student c)...and discover as a group what happens to each of these numbers as they add one more.

Whole Number Addition: To write missing sums for addition examples in vertical or horizontal form.

How to: 1. Students will complete assigned addition problems and go to a calculator center in one corner of the room. They should take only a red pencil to the calculator center and check their work with it. This procedure gives them immediate feed-back on the accuracy of their everyday assignments.

2. If the room has a "grocery store" have each student "buy" products at the grocery store and write his own sales slip example:

3-30-77	
meat	5¢
milk	7¢
apple	10¢
total	_____

I estimate _____

He can then estimate his bill on opposite side. A "checker" can use the calculator to compute bill.

K-3 (cont'd)

3. Use a simple code: A B C D E . . .
1 2 3 4 5 . . .

Have each student figure out the "value" of each letter of his first name, example:

18 + 1 + 11 + 11 + 24 =
S A L L Y = _____

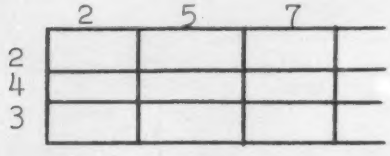
and find answer on calculator: 18 + 1 + 11 + 11 + 24 = 65

Use 65 for name and write number autobiography for display in classroom.

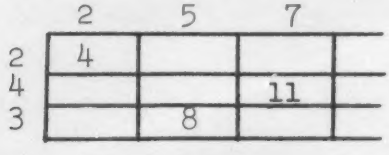
MY NAME IS 65.
I have 2 + 1 sisters,
I have 1 + 0 dogs,
I have 0 + 0 brothers, etc.
Who am I?

Display baby picture

4. Explain addition charts carefully:

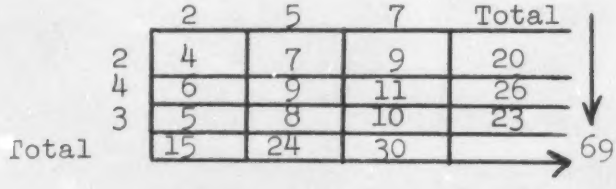


Have each student fill in chart - example:



2 + 2 = 4 3 + 5 = 8 4 + 7 = 11

Filling in each box. When each box is filled in have students check work by adding up each row that they have placed in box - both horizontally and vertically. Example:



Add only numbers that students have written, not the addends that the teacher has selected. If all addition is correct, the totals from the vertical addition should equal the totals of the horizontal column addition. Discuss why this should be so.

K-3 (cont'd)

Subtraction: To write the missing addend of a subtraction sentence written in either vertical or horizontal form.

How to:

1. Check daily work such as:
   a. workbook pages
   b. board work
2. Check weekly projects such as balance on an imaginary "check book"
3. Given a sum of "money" students may "buy" objects from a catalog. He may check his balance two ways:
   a. add up all expenses and deduct from balances.
   b. deduct each expenditure as it is made.
   Discuss why answers are the same.

Number Order: To write the number that comes between two consecutive even or odd numbers.

How to: The student can check his/her answer on the calculator by keying in the smaller number and adding one. Adding 1 again will give the larger number.

Addition-Multiplication Relationship: To recognize the relationship between addition and multiplication.

How to: Use two calculators. Have student select a multiplication problem (1 digit × 1 digit) or (2 digit × 1 digit).

Example: $15 \times 2 =$

1. Calculator II - Use "+" to add 2, 15 times.
2. Calculator I - Use the "×" key.
3. Write problems on board while doing them.
4. Do this with several problems until most students see the relationship between adding and multiplying.
5. Have students put another problem on board using numbers and dots and solve on calculator for those slower students who still need reinforcement.

Example: $7 \times 2 = 14$

$2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$

6. Show similar relationships between division and repeated subtraction.

Zero in Addition or Subtraction: To discover the role of zero in addition or subtraction.

How to: This can be a small group, whole class or individual learning activity.

1. Add 0 to as many numbers as you can think of.
2. Write the answers down. Then do the problems on the calculator. Write down the answers.

3. What happens in every problem?
4. Does it matter if the 0 is before the other number or after the other number?
5. Write a rule for adding 0 to another number. "When I add 0 to another number I always get ..."
6. For subtraction carry out a similar sequence of activities.

Zero in Multiplication: To discover the role of zero in multiplication.

How to: Same type activities as with addition except extend the exercise to more precise notation. After students have written rule in words show them that N can stand for any number and that $N \times 0 = 0$
$0 \times N = 0$
is a way to write the generalization.

Multiplying by Ten: To discover a short cut for multiplying by ten.

How to:
1. Have students think of about 15 numbers.
2. He should write them down and multiply each by 10: 6×10 100×10 178×10 etc.
3. Do all of the problems on the hand calculator. Note each answer: $6 \times 10 = 60$
$100 \times 10 = 1000$ etc.
4. What can the student notice happens when he multiplies 10 and another number?
5. Does it make any difference if 10 is first in the problem or last?
6. Ask him to write a rule for multiplying 10 and another number.

Commutative Property for Addition and Multiplication: To discover that $a + b = b + a$ or $a \times b = b \times a$.

How to: Have students use calculators to figure out several pairs of problems (in either adding or multiplying) and switch around the numbers.

Example:

	$3 + 1 =$	$78 + 39 =$
	$1 + 3 =$	$39 + 78 =$
	$6 + 2 =$	$362 + 145 =$
	$2 + 6 =$	$145 + 362 =$
or	$4 \times 5 =$	$20 \times 31 =$
	$5 \times 4 =$	$31 \times 20 =$

Discuss results. What rule can they write for addition or multiplication?

K-3 (cont'd)

Pattern Discovery: To discover the pattern of a sequence of numbers.

How to: Have students continue a number pattern.

Example: 2 5 8 11 14 __ __ __ __ __ 32

Student can check it on the calculator, i.e. count by threes to see if 32 occurs in the correct spot.

Place Value and Base: To produce the standard form of a number when given a non-standard form.

How to: 1. The teacher dictates a number in "tens", "ones" form and the student displays the number on the calculator. For example given "seven tens, three ones", the student displays "73".
2. Give "8 thousands, 4 hundreds, 6 tens, 5 ones" in written form. The student displays 8465.
3. Give "7000 + 200 + 5" in written form. The student displays 7205.
4. Give "3 hundreds, 15 tens, 4 ones" in written form. The student displays 454.

Checking Center:

A "daily work checking center" using hand calculators can be well worth the small amount of time to set it up. Firm rules need to be established to make it effective. One successful early-elementary center was developed in this way:

1. Four able students were taught to use the hand calculator to check workbook and board work activities.
2. ONLY RED PENCILS WERE ALLOWED AT THE CHECKING CENTER.
3. As each student completed his work, which was done in regular pencil, he went to the checking center and his work was checked on the calculator by a checker.
4. Any errors on the sheet were marked incorrect and he copied them in red pencil on another piece of paper, returned to his desk to do them again. His original paper stayed at the checking center for the teacher to collect.
5. After five days as checkers, the four students taught four more how to use the hand calculators and they became checkers for five days.

GRADES 4-8

A. Number-Numeration

Place Value: To use the calculator to solve place value problems.

How to:

1. Solve equations such as:
   $936 = n + 30 + 6$ or $3495 = 3000 + 400 + n + 5$.
   For any given numeral the student can record the number in expanded notation. The calculator can then be used to sum up the addends to give the original number.

2. Use the calculator to write the correct numeral for expressions such as: six thousands, five hundreds, two tens, and three ones or three thousand five hundred seven. After the students exhibit basic understanding of place value with counting groups of 10's, 100's, 1000's etc., the calculator can be used to help students to translate word names for specific numbers such as above. Students then record the numeral names for the given numbers.

3. Use the calculator to find the number that is 1000 less than numbers like 6254 or 9000. The calculator lends itself to teaching the concepts of order and "less than--greater than." Students very quickly learn to verify their answers by adding 1000 to their choice. Many problems can be designed to generate greater numbers as well. For example, "Show on your calculator the number 100 greater than 4821" (4935 would be harder). Check your answer!

B. Whole Numbers - Basic Operations

Addition: To use the calculator in mastering addition skills and concepts.

How to: Use the calculator to check answers for exercises such as:

9	296	7	7302
+ 7	+ 348	6	2986
		+ 8	+ 8547

Calculators can best be used to reinforce concepts and computational algorithms. In most situations students should use the calculator to verify the summation of columns of digits rather than to find the sum all at once. This procedure helps students apply and understand the concepts of addition, place value and the computational algorithm more completely.

4-8 (cont'd)

Subtraction: To master subtraction skills and concepts.

How to: 1. Use the calculator to check answers for exercises such as:

11	24	97,683
- 3	- 16	- 69,794

Problems can be worked on pencil and paper and corrected with the calculator. In problems where regrouping or borrowing is necessary, the calculator can be used to verify the regrouping. Students should estimate answers, then perform the subtraction operation either on paper or with the calculator.

2. Find the missing addends by solving the inverse subtraction problem:

A. $\square + 5 = 9$	B. $n + 8 = 14$
$9 - 5 = \square$	$14 - 8 = n$

The calculator is an aid to understanding inverse operations. Students can see the inverse relationships between addition and subtraction readily with a calculator because the student actually does the addition and subtraction by punching the appropriate operation button on the calculator. This is more of a hands-on learning experience than doing the same problems with paper and pencil only.

Open sentences become easier to do because students get immediate feedback. Answers can be corrected more easily. Students enjoy these drills more with a calculator.

Multiplication: To master multiplication skills and concepts.

How to: 1. Use the calculator to check answers of the following:

8	32	452
x 4	x14	x596

2. Use your machine to fill in the place holders in examples such as:

45	135	129	475
8□	6□	x 243	x 369
+ □9	+ □4	387	4□75
161	245	51□□	28□□0
		25□□□	14□□□□
		31347	175275

4-8 (cont'd)

The calculator can be used to reinforce the teaching of the multiplication process by having the students use the calculator to find the partial products and performing the addition of the partial products. The answer can be verified by using the calculator to multiply the factors together to obtain the product.

3. Complete the table with your calculator:

×		68	
19	665		
68			3876
	315		

Tables such as above can be designed to test basic understanding of the factor × factor = product relationship. In order to complete the table the student must demonstrate an understanding of the inverse relationship of multiplication and division. With the calculator the student can perform the indicated operations to complete the table. Students can be asked to complete the table with pencil and paper and verify results with the calculator.

4. Multiply $1 \times 2 \times 3 \times 4 \times 5 \times 6 \ldots . 12 = \square$ (This product is called 12 factorial and is written 12!)

Factorials are very quickly understood when taught with a calculator. If you have a calculator with a factorial key, students can be shown the value of such a key after they have performed the factorials long-hand and/or with a basic calculator.

5. Find the value of the following:

(2^3 means $2 \times 2 \times 2 = 8$)

$3^2 = \square$ $\quad 5^3 = \square$

$5^3 \times 5^2 = \square$ $\quad 4^6 - 4^2 = \square$ $\quad 2^{13} = \square$

Stress that $2n \neq 2^n$. For example, $2 \cdot 6 \neq 2^6$

Many opportunities for increasing understanding of exponential notation are available with the calculator. You can have students see the difference between $2 \times 2 \times 2$ and 3×3 to emphasize the mean-

4-8 (cont'd)

ing of 3 in 2^3. Rules can be motivated by calculating 5^3, 5^2 and 5^5 on the calculator or 4^6, 4^2 and 4^4 for $4^6 \div 4^2 = 4^4$. Later a power raised to an exponent, $(3^2)^4$, can be investigated as can $3^{(2^4)}$. Here the calculator can aid understanding!

6. Use your calculator to evaluate the equations:

$\square \div 17 = 408$, $(8 \times \square) + (5 \times \square) = 104$

$(8 \times \square) + (5 \times \Delta) = 102$, $\square \times \square = 2122849$ (or 212.2849)

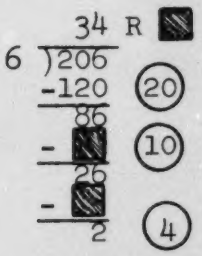
Students soon find out that they have great confidence in their ability to solve equations when much of the arithmetic drudgery is removed. They also realize that the calculator only calculates. Students become aware of the need to understand the equation-solving process in order to select the proper order of button selection on the calculator. The final equation can be used to encourage exploratory activity with the calculator.

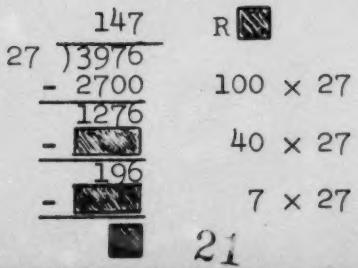
Division: To master division skills and concepts.

How to: 1. Use the calculator to check answers on the following:

$81 - 9 = n$ $\quad 3\overline{)94}$ $\quad 5\overline{)4865}$

2. Fill in the missing numbers using the calculator.





4-8 (cont'd)

For the operation of division with the calculator you get answers, that's all! The teacher must teach the division algorithm. However, the teacher can use the calculator to teach the long division process by having students perform each step of the long division problem with the use of the calculator as shown in the examples. The last example illustrates finding the greatest multiple at each step. It is closer to the final algorithm than the first two developmental examples.

The quotient appears as a decimal on most calculator displays. This requires that the "remainder form" and the "decimal form" of division answers be discussed and related. Perhaps curricular revisions will be needed to introduce decimal notation for fractions earlier.

3. Use the calculator to find the missing factors by solving the related division problems:

$n \times 6 = 30$ $\qquad$ $n \times 7 = 63$

$30 \div 6 = n$ $\qquad$ $63 \div 7 = n$

The inverse principle is a very important concept for students to discover. The calculator offers students the opportunity to quickly verify the relationship between the operations of multiplication and division. For example: students soon discover with appropriate amount of practice that if $4 \times 3 = 12$ and $3 \times 4 = 12$ then $12 \div 4 = 3$ and $12 \div 3 = 4$. One problem to watch for is the discrepancy between, say 5 and $(5 \div 7) \times 7$. On the calculator they tend not to be equal even though they are theoretically equal.

4. Use the calculator to determine if the following numbers are even or odd.

38 $\quad$ 653 $\quad$ 1,692 $\quad$ 29 $\quad$ 458 $\quad$ 8,429 $\quad$ 11,136

With the calculator, students can test even numbers by quickly dividing by 2. The divisibility test can be performed more quickly and efficiently with a calculator.

5. Solve the following and check your answer using the calculator:

$$\begin{array}{r} 37 \text{ R } 2 \\ 9\overline{)335} \end{array} \qquad \begin{array}{r} 37 \\ \times 9 \\ \hline 333 \\ +\ 2 \\ \hline 335 \end{array}$$

4-8 (cont'd)

This demonstrates to students the principle $a = (b \times q) + r$

Checking is important to ensure correct answers and to emphasize the relationship between multiplication and division. The use of the calculator should encourage students to check divisions.

6. Find the average of the following sets of numbers:

   a) $5 + 9 + 10 + 4 = 28$

   b) $2 + 3 + 10 + 9 + 11 = 35$

Some calculators have an averaging key. Consult the manual on how to use it. However, students can obtain the average of these problems without use of an averaging key.

Estimation: To become skillful in estimating results of computation.

How to: First estimate the sum and differences, and then use the calculator to check your estimation:

1. $5812 + 4406 =$ _____
2. $864 - 231 =$ _____
3. $479 \times 24 =$ _____
4. $8153 \div 41 =$ _____

Traditionally students have resisted estimation work. The calculator may help here since the check is done using the machine rather than by hand. Some work with rounding is needed for estimation skill as well as mastery of facts and products of powers of tens.

Order Relations: To order numbers and number expressions.

How to: 1. Select the greatest number in each row. Use your calculator to check your results.

(a)	(b)
19×31	32×19
$889 - 496$	$889 - 496$
$380 - 20$	$400 - 19$
$2 \times 20 \times 40$	$2 \times 19 \times 30$

2. Insert the correct symbol: $>$, $<$ or $=$ to make the statement true. Then check the results with the calculator.

$45 \times 31 \square 43 \times 31$

$178 - 6 \square 178 - 7$

$127 \times 428 \square 428 \times 127$

$32 \times (41 + 82) \square (32 \times 41) + (32 \times 82)$

4-8 (cont'd)

The calculator is helpful in teaching the greater than ($>$), less than ($<$) concepts. Here again students should be asked to estimate answers, record estimated answers, perform the required operations with a calculator. If there is a need to have students do the work on paper, they should do so and correct their work with a calculator.

3. Use the calculator to find the largest number from the set (10, 20, 30, 40, 50, 60, 70, 80, 90) that will make each sentence true.

   a) $n \times 9 < 387$

   b) $n \times 7 < 546$

   c) $n \times 9 < 747$

4. Use the calculator to find the largest whole number that makes each sentence true.

   a) $n \times 6 < 493$ $\qquad$ $n \times 6 + \square = 493$

   b) $n \times 8 < 257$ $\qquad$ $n \times 8 + \square = 257$

   c) $9 \times n < 329$ $\qquad$ $9 \times n + \square = 329$

Number Properties: To learn the fundamental properties of the operations on whole numbers.

How to: 1. Find the missing numbers with your calculator:

$589 + \underline{\qquad\qquad} = 589$ $\qquad$ $17 + 94 = \square + 17$

$(49 + 24) + 37 = 49 + (24 + \square)$

$587 \times \square = 587$

$9 \times (38 + 72) = (9 \times 38) + (9 \times \square)$

Number properties are important ideas of mathematics for students to understand. After students have had sufficient experiences with number properties in the classroom without the use of a calculator, they can be assigned lessons like the above to further improve their understanding of the following properties:

a) addition of zero ($5 + 0 = 5$)

b) multiplication by one ($15 \times 1 = 15$)

c) order properties:
   commutativity for addition ($5 + 6 = 6 + 5$)
   commutativity for multiplication ($8 \times 7 = 7 \times 8$)

d) grouping properties:
   associativity for addition ($(7 + 6) + 4 = 7 + (6 + 4)$)
   associativity for multiplication ($(9 \times 4) \times 8 = 9 \times (4 \times 8)$)

4-8 (cont'd)

2. Use the calculator to solve the following:

a) $24 + 36 = \square + 24$

$60 = \square$

b) $59 \times 83 = \square \times 59$

$4897 = \square$

3. Use the calculator and solve the following problems:

$(1 + 2) + 3 = 1 + (2 + 3)$

$\square + 3 = 1 + \square$

$\square = 6$

$(4 \times 6) \times 3 = 4 \times (6 \times 3)$

$\square \times 3 = 4 \times \square$

$72 = \square$

Factors and Factoring: To master the concepts and skills associated with factoring and factors.

How to: 1. Use the calculator and list all the factors for the following:

a) 25 b) 50 c) 100

2. Which of the following numbers are factors of the first number:

18 ⟶ 1,2,3,5,6,8,9,12,18
37 ⟶ 1,5,7,10,27,37
45 ⟶ 1,3,5,7,9,10,15,20,45

3. Find the greatest common factor (G.C.F.) of:

a) 28 and 35
b) 18 and 24
c) 16 and 24
d) 18 and 27

Use the calculator to generate factors. Examine factors to find greatest.

4. Find the least common multiple (L.C.M.) of:

a) 28 and 35
b) 18 and 24
c) 16 and 24
d) 18 and 27

Use the calculator to find sets of multiples of each number. Find the least such common multiple. This activity can be used to help develop concept of L.C.M. Other means may then be taught to calculate the L.C.M. more efficiently. Ask students if there is any way to relate L.C.M. and G.C.F.

4-8 (cont'd)

Patterns: To investigate patterns of sequences of numbers.

How to: Produce these sequences on your calculator:

a) 2,4,6,8,10,...,22,...
b) 1,3,5,7,9,...,21,...
c) 5,10,15,20,25,...,55,...
d) 2,0,-2,-4,-6,...,-16,...
e) 1,2,4,8,...,512,...
f) 1, 0.5, 0.25, 0.125,...,0.0039062,...
g) 1,4,9,16,...,100,...
h) 12,9,6.75,5.0625,...,0.9010161,...

Encourage students to seek a pattern. When they think they have a pattern, have them test it on the calculator. Suggest that students create new patterns of their own.

C. Common Fractions and Decimal Fractions

Equivalent Fractions: To determine if two fractions are equivalent (name the same number).

How to: 1. Use the calculator to determine if the following fractions are equivalent:

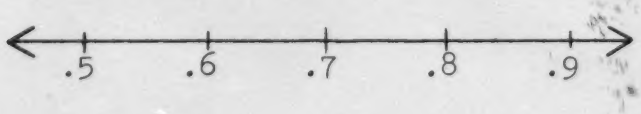
$\frac{1}{2} \bigcirc \frac{3}{4}$ $\frac{6}{8} \bigcirc \frac{3}{4}$

$\frac{3}{7} \bigcirc \frac{9}{21}$ $\frac{5}{9} \bigcirc \frac{25}{36}$

One method is to find a common denominator and then compare the numerators. With the calculator as an aid, the easiest common denominator to use is the product of the given denominators. Using this method, 8 is the common denominator for $\frac{1}{2}$ and $\frac{3}{4}$. Two other methods are changing to decimal form and "cross multiplying". Both require more conceptual and skill development than the method suggested above.

2. Locate the following on the number line. Use your mini-calculator.

$\frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{7}{9}$



4-8 (cont'd)

Students need instruction in the decimal equivalents of common fractions. This example, as well as those that follow, requires the application of the skill of converting common fractions to decimal names which must be mastered first.

3. Insert $>$, $<$ or = in the circles. Use your calculator.

$\frac{3}{4} \bigcirc \frac{9}{12}$ $\frac{30}{60} \bigcirc \frac{51}{91}$ $\frac{22}{43} \bigcirc \frac{203}{404}$

4. Represent these as decimals. What do you discover?

$\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$

5. Use your calculator to compute:

$\frac{3}{7} + \frac{11}{31} = \square$

Basic Operations with Decimals: To master the concepts and skills of operating on decimals.

How to: Place the decimal point in the answer. Check your answers with your calculator.

$2.1 + 3.2 + 4.1 = 940$ $13 - 6.97$

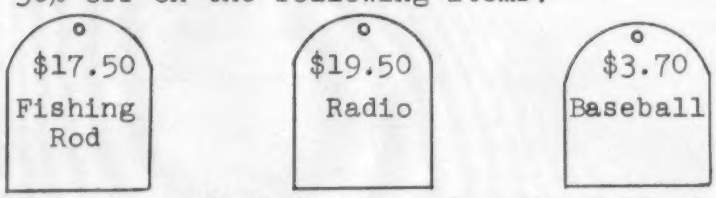
$12.96 - 4.83$ $5.49 \times 3.2 = 175680$ $.19\overline{)239.5}$

The skill is developed elsewhere. Here emphasize placement of decimal points. Use calculator as feedback device.

Per Cent: To solve per cent problems.

How to: 1. Use your calculator to determine the sale price of these items.

30% off on the following items:



2. Use your calculator to solve the following problems:

a) 15% of 125
b) 25% of 630
c) 50% of 290

4-8 (cont'd)

d) 65% of 850
e) What percent of 14 is 25?
f) 15% of what number is 85?
g) What number is 75% of 43?
h) 37 is what percent of 287?

If your calculator has no percent key, students need to know how to convert per cents to decimals. Teach this before you do the applied exercises above.

D. Integers

Basic Operations: To master the skills and concepts of operating on integers.

How to: 1. Find the additive inverse of each number with the assistance of your calculator.

a) -24 b) 17 c) $\frac{1}{2}$ d) $-\frac{2}{3}$

Each calculator has its way to change the sign of a number. Help students to learn how to do this.

2. Use your calculator to find the value that makes it true!

a) $^{+}4 + {}^{-}5 =$
b) $^{-}8 + {}^{+}10 =$
c) $^{-}12 + {}^{-}6 =$
d) $^{-}4 \times {}^{+}3 =$
e) $^{-}8 \times {}^{-}5 =$
f) $^{-}72 \div {}^{-}12 =$
g) $^{-}12 - (^{-}6) =$
h) $12 - (^{-}5) =$
i) $36 \div {}^{-}12 =$
j) $^{-}43 \div 12 =$

Estimate answers first--including sign of result. Check the estimates by using the calculator.

E. Supplementary Materials.

Function Machines: To find the output for function machines.

How to: 1. Use the calculator to solve the following:

Add 7 if odd Add 8 if even	
Input	Output
5	
6	
7	
8	

7 or less: Add 8 >7 sub. 8	
Input	Output
4	
15	
10	
7	

4-8 (cont'd)

2. Use the calculator to select the correct function rule:

Input	Output
4	10
7	13
9	
	14

Input	Output
2	6
5	12
3	8
9	20

Magic Squares: To complete magic squares.

How to: Complete the following magic squares.

4	9	
3	5	7
8		

5		
13	8	
6		11

Note: all rows, columns, and diagonals must have the same sum.

Pythagorean Theorem: To discover the Pythagorean relation and to master its use.

How to: 1. Give several right triangles with measures of sides a,b, and hypotinuse c. Use the calculator to complete the table.

a	b	c	a^2	b^2	c^2	$a^2 + b^2$
3	4	5				
5	12				169	
	24	25	49			
	21	29				841
11		61				
8	15					

How are the numbers in the last two columns related?

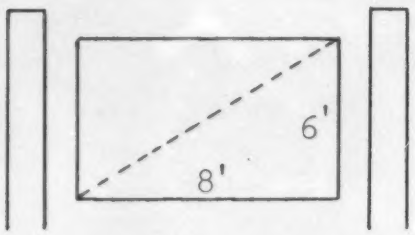
2. Give several triples of numbers. Use the calculator to determine if they are Pythagorean Triples.

a) 3,4,5 b) 4,5,6 c) 6,6,10 d)25,7,24

3. Solve practical verbal problems.
(1) Find the distance across the swamp from point A to point B.

4-8 (cont'd)

(2) How long a piece of wood would you need to make a diagonal brace for the gate below?



Pascal's Triangle: To learn a computing formula for terms in Pascal's Triangle.

How to: After introducing Pascal's Triangle

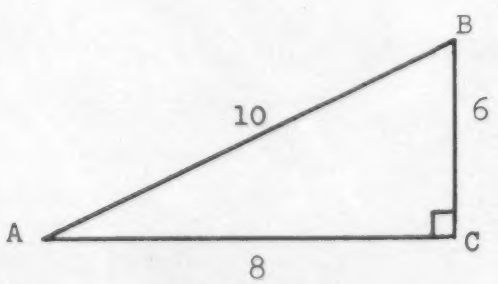
1 1
1 2 1
1 3 3 1
1 4 6 4 1,

introduce the computing formula $\frac{n!}{(n-r)!r!}$ where n gives the row in the triangle, r + 1 is the term number, and $4! = 4 \cdot 3 \cdot 2 \cdot 1$. For example, to find the value of the third term in row 4, you calculate $\frac{4!}{(4-2)!2!} = 6$ on your calculator. Also sum each row and seek pattern of sums.

Trigonometric Ratios: To use the trigonometric ratios.

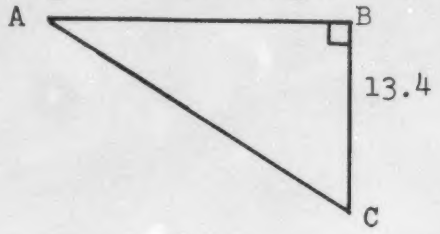
How to: After the basic definitions of the ratios are learned, the calculator can be employed in a variety of ways.

1. Find sin A, and cos B.

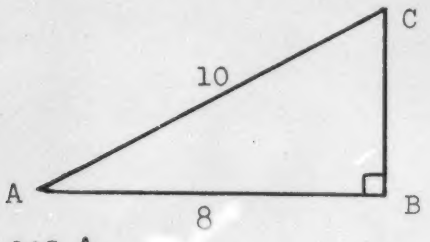


4-8 (cont'd)

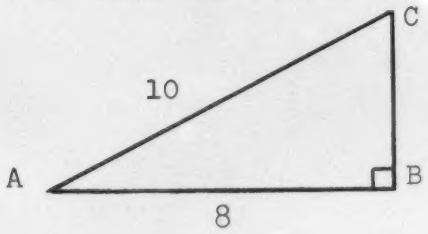
2. Find the length of line segment $\overline{AC}$ if sin A = .67



3. Find tan A. (Find BC by using Pythagoras)



4. Find the cos A.



5. Demonstrate the reciprocal relationship among trigonometric functions. For example: $\csc\Theta = \frac{1}{\sin\Theta}$, $\cot\Theta = \frac{1}{\tan\Theta}$.

Proportions: To solve proportions.

How to: With a calculator we do not have to be so careful in picking the numbers we use in proportions. By calculating the decimal equivalents, the student can solve proportions that involve mixed numbers. For example:

$$x : 3\tfrac{1}{2} = 2\tfrac{3}{4} : 5\tfrac{3}{5}$$

would be transformed to:

$$x : 3.5 = 2.75 : 5.6$$

which would be solved as: $x = \frac{(3.5)\cdot(2.75)}{5.6}$

4-8 (cont'd)

Intuitive Limit Concept: To introduce the idea of a limit.

How to: Even though the limit concept is not standard content in the early secondary curriculum, the calculator gives us the computational ability to examine situations that involve limit ideas. Suppose you have a box that shrinks with every use. If the box holds 10 bushels, can you empty a truck containing 25 bushels with it?

n	capacity	amount left in truck
1	10	15
2	5	10
3	2.5	7.5
4	1.25	6.25
5	.625	5.625
6	.3125	5.3125
7	.15625	5.15625

If we extend the table beyond this point we can see that the amount being removed is almost zero and therefore the amount remaining in the truck does not dinimish much, and in fact we would never unload the truck. Other examples can be set up that are similar to this one which will illustrate the point that under some circumstances a procedure is limited in what it can do. (If the process is carried too far, calculator overflow or underflow may cause errors.)

Square Roots: To find square roots using the guess-divide-average method.

How to: Using only the basic function keys of the calculator we can generate a square root. Consider this: Find the square root of 34 to three decimal places.

step		
1	guess	34 is smaller than 36 therefore its square root is smaller than 6, say 5.8
2	divide	34 divided by 5.8 is 5.862 to three decimal places
3	average	5.8 plus 5.862 divided by two is 5.831
4	divide	34 divided by 5.831 is 5.831, rounded to three decimal places.
5	multiply	$5.831 \times 5.831 = 34.000_____$.
6	stop	We have found the square root to the stated accuracy. We can stop when the divisor and the quotient agree to the number of decimal places required.

4-8 (cont'd)

Use your calculator to find the following square roots to three places.

a. 58 b. 73 c. 27 d. 85

Games and Puzzles: To have fun.

How to: 1. Enter this number 1 2 3 4 5 6 7 8 9. Multiply by any single digit number except zero. Now multiply the result by 9. Try this procedure for several one-digit numbers. What do you discover?

2. Palindromes

Palindromes are words or numerals that read the same forward or backward like "noon." Other examples are: WOW, RADAR, TOT, LEVEL, 121, 252 and 18181.

The following activity deals with (numbers) numerals that can be turned into palindromes.

a) Write any numeral.
b) Reverse the digits and add.
c) If the result is not a palindrome, reverse the digits again and add to the previous result.
d) Sometimes this process must be done several times before you obtain a palindrome.(3)

Example: Take 156 Reverse digits: 651
Add: 156 + 651 = 807
Then reverse digits: 807 + 708 = 1515
Then 1515 + 5151 = 6666 (palindrome)
Now try: 89

3. Use your calculator to find all pairs of numbers that make the equation true.

a) $(7 \times \square) + (5 \times \Delta) = 110$ (solutions: (0,22), (5, 15), (10, 8) and (15, 1).)

b) $(6 \times \square) + (9 \times \Delta) = 108$ (7 solutions)

c) For button pushing whizzes.

$5 \times \bigcirc + (3 \times \square) + (2 \times \Delta) - 100$ (over 100 solutions)

---

(3) Whether all numerals produce palindromes is an unsolved question. See "202 = Bob and 414 = DID: An Investigation of Palindromes Produced By 3-digit Numbers," by Maryjo M. Nichols, Mary P. Freeman, and Anna Marie Wiringer, Mathematics in Michigan, (XVI: 3), January, 1977.

GRADES 9-12

Pre-Algebra and Algebra

The following topics are selected for pre-algebra and algebra classes. No claim for completeness is made. Also no assumptions about calculator sophistication are made. You may need to modify suggestions to fit your machine.

In these illustrations we give the concept or topic name and one or more examples which illustrate the type of activity students may engage in. These are considered as illustrations to stimulate thinking in relation to the classroom situation and the students.

TOPIC	ILLUSTRATION
Order of Operation	Rewrite the following formula so it can be easily done on your calculator. $M = \frac{d - f}{f}$ Answer: $M = (d - f) \div f$
Order of Operations and Properties of numbers	Rewrite the following problem so it can be done on your calculator in one sequence. Check your result. a) $[32 \div (6 + 2)] \times 5$ Ans: 6 ⊕ 2 ⊜ (Recip.) ⊗ 32 ⊗ 5 * b) $(3 \times 5) + (3 \times 4)$ Ans: $(5 + 4) \times 3$ 5 ⊕ 4 ⊜ ⊗ 3 c) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ Ans: b ⊗ d ⊜ (Recip.) × ⓐ × ⓒ * d) $a + \frac{b}{c} = \frac{a \cdot c + b}{c}$ Ans: a ⊗ c ⊕ b ⊜ ± c ⊜
Solving Linear Equation	The calculator is especially convenient for solving linear equations by the "cover up method."

* The answers given assume a reciprocal key on the machine. These translations from algebraic notation to a calculator form are essential for effective use of the calculator in algebra-related topics. They should be thoroughly mastered.

9-12 (cont'd)

The teacher covers the expressions.
The teacher asks, "What number is under my hand? Use your calculator to compute the result."
The student finds the answer by using a related fact of arithmetic. He may be asked to write or to read or to check his answer.

Illustrative Example: (Teacher demonstration)

Solve: $\frac{4.4x + 8.8}{72} = 11$

Solution:

Line 1. $\frac{4.4x + 8.8}{72} = 11$

Line 2. $\square = 792$

Line 3. $4.4x + 8.8 = 792$

Line 4. $\square = 783.2$

Line 5. $4.4x = 783.2$

Line 6. $\square = 178$

Line 7. $x = 178$

Try these:

1) $\frac{.03x - 10}{.7} = 8$

2) $6.5(x + 2.5) = 39$

3) $\frac{4.75x + .08(1000)}{1.026} = 50{,}000$

4) $3\frac{1}{7}x = 60$

Variables and Equation Solving

1. In the following formula, replace the letter (h) with twelve different numbers: $12h + 5$. Find the results. Which replacement will give the result 39?
2. A formula to compute the change in a length of copper wire due to temperature change is:

9-12 (cont'd)

$c = .0000\ 17\ \ell t$ where
c is the change in length,
ℓ is the length in meters,
t is the change in temperature, degrees Celsius.
Find the change in length of a 200 meter copper wire when the temperature change is a drop of 20°C. Ans: -0.068

Ordered Pairs

1. Use the formula $y = \frac{1}{2}x + 5$ and fill in the following table:

x	0	1.25	2.7	9.3	11.62
y					

2. Find ten rectangles that have a perimeter of 30 centimeters. Use decimals. Which of these rectangles has the largest area? How many rectangles have a perimeter of 30?

Ratio and Formula Substitution

1. The formula for the breaking strength of a certain type rope is:
   $w = 5000d(d+1)$ where (w) is weight in thousands and d is diameter of rope in inches. What happens to the strength when the diameter is doubled, tripled, etc?
2. A formula to find the height (h) in feet necessary to see x miles out to sea is $h = \frac{2x^2}{3}$. What happens to h as x doubles, triples, etc.?
3. The formula for the volume of a cube is $V = e^3$. How does the volume (V) change as the edge (e) doubles, triples, etc.?

Formula Substitution and Equation Solving

A formula for measuring a typist's speed in words per minute is:

$$s = \frac{w - 10e}{t} \text{ where}$$

s is the speed in words per minute.
w is the number of words typed.
e is the number of errors.
t is the time of the test.

Use your calculator efficiently to solve the following problem:

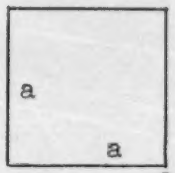
9-12 (cont'd)

700 words 5 errors in 30 minutes.
Ans: s = the opposite of:

$$10 \otimes e \ominus w = \div t$$

Geometric Models of Algebraic Expressions

The geometric model of a^2, $a > 0$, is a square region of side a and area a^2.

Algebraic	Geometric
a^2	

area = a^2

1. Use your calculator to find a^2 when $a = 39.7$
2. Draw geometric models for each of the algebraic expressions.
   a. $a(b + c) = ab + ac$
   b. $(a + b)\cdot(c + d) = ac + ad + bc + bd$
   c. $(x + a).(x + b) = x^2 + ax + bx + ab$
   d. $(a + b)^2 = a^2 + 2ab + b^2$
   e. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ac + 2bc + 2ab$
3. Use your calculator to compute:
   a. $a(b + c)$ when $a = 555$, $b = 467$, $c = 302$
   b. $ab + ac$ when $a = 7\frac{1}{4}$, $b = 3\frac{3}{8}$, $c = 5\frac{1}{2}$
   c. $(a + b).(c + d)$ when $a = 73$, $b = 144$, $c = 88$, $d = 99$
   d. $(x + a).(x + b)$ when $x = 50$, $a = 42$, $b = 28$
   e. $(a + b + c)^2$ when $a = 1.14$, $b = 6.7$, $c = 29.5$

   In c-e, students may be asked to investigate which is faster, adding in parentheses and then multiplying, using $a(c + d) + b(c + d)$, or $ab + ac + bc + bd$. Teams could be formed to race the answers.
4. How would you show geometric models when some numbers are negative?

Percent

Write a calculator sequence to change the following fractions to percents:

a. $\frac{2}{3}$ Ans: $\frac{2}{3} = \frac{x}{100}$ $2 \otimes 100 \oplus 3$

b. $3\frac{1}{4}$ Ans: $3\frac{1}{4} = \frac{x}{100}$ $3\frac{1}{4} \times 100 = x$

$(\frac{12}{4} + \frac{1}{4}) \times 100 = x$

$4 \otimes 3 \oplus 1 = \oplus 4 \otimes 100$

Multiplication of Decimals

Replace each * with the correct numeral.

$$\begin{array}{r} * *.* * \\ \times \quad 0.5\ 6 \\ \hline * * * * \\ 6\ 1\ 7\ 0 \\ \hline * * * * * \end{array}$$

Factoring

1. Find all the factor pairs for the following numbers: 72, 144, and 288.
2. A perfect number is a number whose proper factors add to itself, i.e. $1 + 2 + 3 = 6$. Find a perfect number under 50. Show that 496 is a perfect number.

Square Numbers

Use a square with an array of dots n per row in n columns. The total number of dots is the product of n^2. Draw the arrays.

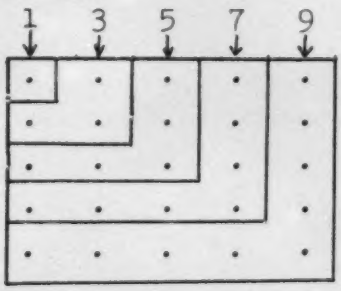
Observe that the square arrays can also be represented as the sums of consecutive odd numbers of the form:

$$n^2 = 1 + 3 + 5 + \ldots + 2n - 1$$

Separate each array drawn to show that each is a sum of odd integers.

This may be illustrated graphically on the geoboard as:

$\{3^2 = 9 \quad \{2^2 = 4 \quad \{1^2 = 1 \rightarrow$



9-12 (cont'd)

Complete:

a. $1+3+5+7+9+11+13+15+17+19+21+23+25=(?)^2$

b. $1 + 3 + \ldots + 999 = (?)^2$

c. $1 + 3 + 5 + \ldots + 2k - 1 = (?)^2$

d. m and (m + 1) are consecutive integers. The difference of their squares is 51. Find m and (m + 1).

e. Use your calculator with repeated subtraction to do the following:

$2209 = 1 - 3 - 5 - 7 - \ldots - (2k - 1) - 0$

f. $2209 = (?)^2$

g. Find two squares whose difference is 647.

Greatest Common Factor

Find the greatest common factor of 404, 696 and 1970.

Least Common Multiple

Find the least common multiple for 15, 75 and 50.

Prime Numbers

With the aid of your calculator and a table of primes, explore the following question. Is every even number greater than 2 the sum of 2 primes? (This is an unsolved problem.)

Square Numbers

Solve the following equations by trial and error.

1. $x^2 = 16$
2. $x^2 = .16$
3. $x^2 = .0016$
4. $x^2 = 1.6$

Two Equations - Two Unknowns

Introductory activity: Solve the following problems with your calculator. Use trial and error. Sam has 21 stamps. Some cost 13 cents and some cost 8 cents. Total value of stamps is $2.28. How many of each are there?

Use of a Table to Solve Story Problems

Two hikers, John and Mary, left the camp for a mountain side. Mary left at 8:00 and averaged 4.5 m.p.h. John left at 9:00 and averaged 6.0 m.p.h. At what time did John catch up to Mary? Fill in the following table with the aid of your calculator:

Time	8:00	9:00	10:00
Mary's distance	0	4.5	
John's distance	0		

Game

Find three numbers whose sum equals their product.

Formula Substitution And Ratio

1. The following formula can be used to find horsepower for an automobile.

   $h = 0.4nd^2$ where h is horsepower
   n is number of cylinders
   d is diameter of piston in inches

   a. Find the horsepower of an 8-cylinder engine that has a 2.75 inch piston diameter.

   b. Use the calculator to find out what happens to the horsepower as the diameter changes, i.e. doubles, triples, etc.

2. The following formula can be used to find stopping distance of a car on dry pavement.

   $d = 1.1s + .05\ s^2$
   d is stopping distance in feet
   s is car's speed in m.p.h.

   Use the calculator to find out what happens to d as s changes.

3. The formula, $t = \sqrt{\frac{d^3}{216}}$ can be used to estimate how long a storm will last. (t) is the time in hours and (d) is the diameter of the storm in miles. Use the calculator to find out what happens to t as d changes.

4. The formula, $s = \sqrt{24\ d}$ and $s = \sqrt{12\ d}$ can be used to determine the speed of automobiles that have skidded to a stop on dry and wet concrete respectively. (s) is the speed in m.p.h. and (d) is the distance the car skidded in feet. Use the calculator to find out how "s" changes with "d", and how speed will differ on wet and dry concrete for the same skidding distance.

(9-12 cont'd)

The following examples are more applicable to algebra classes than to pre-algebra classes.

Formula Substitution and Order of Operations

1. The formulas $n^2 - n + 11$, $n^2 - n + 41$, and $n^2 - 79n + 1601$ produce primes. (n) is a counting number. Use the calculator to find out if each formula produces primes for every value of (n).
2. The formula $h = vt - 16t^2$ is used to relate the height (h) of a projectile to time (t) in flight. The object has initial velocity (v). Use the calculator to find the change in (h) as (t) changes.
3. The speed of a skydiver after 2 seconds of falling can be determined by $S = 64 + 16t$. (S) is speed in feet per second and (t) is time in seconds. Use the calculator to find the speed in miles per hour of a diver who has fallen for 6 seconds.
4. The formula $s = \frac{2\pi r}{t}$ is used to find the speed (s) of a rotating object. (r) is the radius and (t) is time. Three statues on a merry-go-round are 36, 30, and 24 feet from the center. The merry-go-round rotates once in .5 minutes. Find the speed of each statue.

Rational Expressions

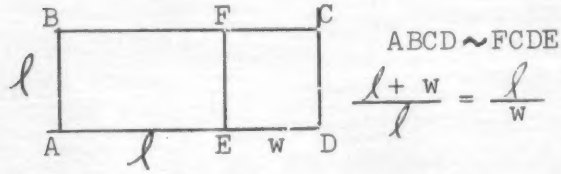
1. The formula $\frac{x}{2\ell - x} = \frac{R_1}{R_2}$ is used to find a defect in a telephone line.

ℓ = length of wire
x = distance from point A to defect
R_1 and R_2 are resistance readings made at A. Find (x) if $R_1 = 500$ ohms, $R_2 = 1200$ ohms, and $\ell = 50$ miles. Rewrite the formulas so it can be done efficiently. Use your calculator.

2. The formula $d = \frac{s^2}{30f} + 1.1s$ is used to relate stopping distance (d) in feet to speed of a car (S) in m.p.h. (f) is the coefficient of friction. For dry concrete f is 0.8. A car traveling 55 miles per hour on dry concrete would have what stopping distance?

9-12 (cont'd)

3. A Golden Rectangle is a rectangle that can be partitioned into a square of side (ℓ) and a rectangle with dimensions (ℓ) and (w) that is similar to the original. The similar rectangles satisfy the proportion $\frac{\ell}{w} = \frac{w + \ell}{\ell}$. With the aid of your calculator find some rectangles that are Golden Rectangles, or nearly Golden Rectangles. What is the ratio $\frac{\ell}{w}$?



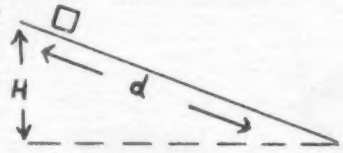
$ABCD \sim FCDE$

$\frac{\ell + w}{\ell} = \frac{\ell}{w}$

Exponents and Quadratic Equations

1. The formula $d = 16t^2$ relates distance (d) in feet to time (t) in seconds of a falling object. Use your calculator to explore the change in d as t changes.

2.



$d^2 = KHt^2$ where t is the time required to travel distance d. K is a constant determined by gravity and friction. Perform an experiment to find K for various materials.

Geometry Through Higher Mathematics

The major goals of geometry, algebra two, trigonometry, senior mathematics, and calculus are not concerned with computational skills. It is therefore assumed that the calculator can be used to do any calculations required in problem material.

(9-12 cont'd)

The type of calculator used in these areas should have functions in addition to +, -, ×, ÷, to be of maximum use. A minimum of these functions would include square, square root and reciprocal, but the trigonometric functions, logarithmic function, and exponential functions are useful additions. Such calculators are now available for reasonable prices.

In this section, as in the last section, no attempt has been made to include all possible topics for which the calculator is useful. Rather examples are included as stimulators to thought. Also included are some interesting number properties which are easily accessible when a calculator is available.

Areas and Volumes

The calculator allows teachers to use better approximations to π and to use rational number dimensions.

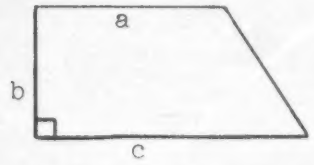
Find the areas of the regions in examples 1-4.

1. Use π = 3.1416

a = 330 ft., b = 25 ft., c = 50 ft.

Ans: 18463.5 sq. ft.

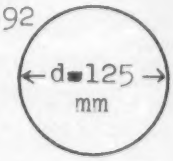
2.



a = 4.295, b = 6.500, c = 7.885

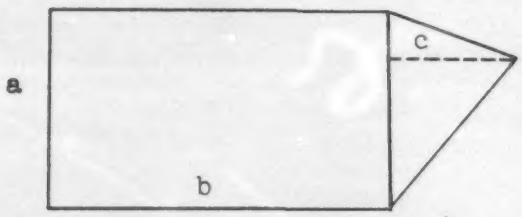
Ans: 39.585

3. Use π = 3.141592



Ans: 12271.843 mm^2

4.

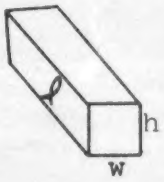


a = 11 in., b = $13\frac{1}{2}$ in., c = $5\frac{1}{16}$ in.

Ans: 176.34 sq. in.

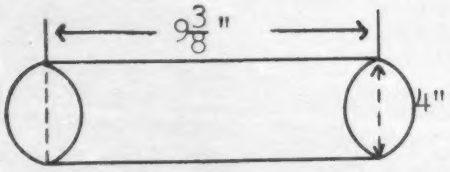
Find the volumes of the figures in examples 5-8.

5. l = 30 cm., w = 17 cm, h = 28 cm



Ans: 14280 cm^3

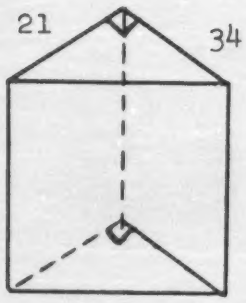
6.



Use π = 3.14

Ans: 117.75 in^3

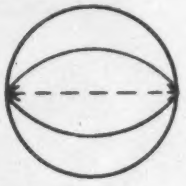
7.



Ans: 12495

9-12 (cont'd)

8. Use $\pi = 3.1416$, $V = \frac{4}{3}\pi r^3$



diameter = 8700 miles

Ans: 3.448×10^{11} cu. mi.

Complements and Supplements

1. The measure of angle A is 7 times the measure of angle B. The angles are supplementary. Find their measures. Ans: 157.5°, 22.5°
2. The difference of one half the measure of $\angle A$ and 7 is 4 times the measure of $\angle B$. If the angles are complementary, find their measures. Ans: $81.\overline{5}^\circ$ and $8.\overline{4}^\circ$

Pythagorean Triples

The following formula can be used to generate families of Pythagorean triples. Generate several families of triples with aid of your calculator.

$$x^2 + y^2 = z^2$$

Use (m,n) where $m > n$

$$x = m^2 - n^2 \qquad y = 2mn$$

$$z = m^2 + n^2$$

i.e. $m = 2$, $n = 1$, $x = 3$, $y = 4$, $z = 5$

Trigonometric Ratios: Reciprocal and Complementary Relationships

Use only your calculator to aid you in filling out the following table. Round off all answers to four places past the decimal.

	SIN	COS	TAN	COT	SEC	CSC
x	.6428		.8391			
90-x	.7660					

Area of Irregular Quadrilaterals

The formula:

$$A = \frac{1}{4}\sqrt{4\,p^2q^2 - (b^2 + d^2 - a^2 - c^2)^2}$$

gives the area of a quadrilateral with the sides, a,b,c, and d and diagonals p and q.

Find A if a = 13.2, b = 11.9, c = 11.0
d = 19.6, p = 20.3, q = 17.5

Ans: A ≐ 168.02

Area of a Triangle

$$A = \sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)}\ ,\ s = \frac{a+b+c}{2}$$

Find the area of a triangle with sides a = 10.8 cm, b = 13.9 cm, c = 15.8 cm

Ans: A ≐ 73.54 cm^2

Flow Charting

Write flow charts so the following problems can be computed on a calculator.

1. $\frac{7-8}{5\times6}$ Note: $\left(\frac{7-8}{5\times6} = (7-8)\times\frac{1}{5\times6} = (7-8)\times\frac{1}{5}\times\frac{1}{6}\right)$
2. $5280\overline{)120{,}000}$
3. $ax + b = c$ Ans: $x = c - b \div a$
4. $.052x + 5280.7 = 7342.8$
   Ans: $x = (7342.8 - 5280.7) \div .052$
5. $5 + 8 \times 2\ (7 + 2.07) \div 3$
   Ans: $[(7 + 2.07) \times 2 \times 8 \div 3] + 5$

Scientific Notation

On a calculator with an 8 digit display do the following computation:

$$215{,}000{,}000 \times 7580$$

Ans: $2.15 \times 10^8 \times 7.58 \times 10^3 =$
$(2.15 \times 7.58) \times 10^{11}$

Converting Units

Set up the following problem so it can be done efficiently on the calculator. Check your set-up by cancelling units. Convert 50 miles per hour to centimeters per second.

Ans: $\frac{50 \text{ miles}}{1 \text{ hour}} \frac{5280 \text{ ft.}}{1 \text{ mile}} \frac{12 \text{ in.}}{1 \text{ ft.}} \frac{2.54 \text{ cm}}{1 \text{ inch}} \frac{1 \text{ hour}}{3600 \text{ sec.}}$

9-12 (cont'd)

Constructing a Table of Values for Graphing Polynomial Functions

The method of nested multiplication
Example:

1. $y = 7x^3 + 5x^2 - 9\frac{1}{2}x + 14$, can be written in the form,

2. $y = ((7x + 5)x - 9\frac{1}{2})x + 14$

Using the **handheld** calculator with the algorithm of nested multiplication gives a convenient procedure for repeating numerous calculations needed in constructing a table of values.

3. Write the following exercises in nested form:

$$y = -8x^2 + 12x + 11$$

$$f(x) = 3x^4 - 2x^3 + 7x^2 - 9x + 8$$

Ans: $y = (-8x + 12)x + 11$

$f(x) = (((3x - 2)x + 7)x - 9)x + 8$

4. Construct a table of values for graphing:

$y = 2x^3 - 3x^2 - 36x + 75$ for the values $x = -5, -4.5, \ldots, 4.5, 5.0$
(Use the method of nested multiplication) Ans: $y = ((2x - 3)x - 36)x + 75$

for $x = -5$,

$y = ((2[-5] - 3) \cdot [-5] - 36) \cdot [-5] + 75$
$= ((-13) \cdot [-5] - 36) \cdot [-5] + 75$
$= (29) \cdot [-5] + 75$
$= -70$

Similarly continue with the other values of x. The following table results.

x	y
-4.5	-6.000
-4	43.000
-3.5	78.500
-3	102.000
-2.5	115.000
-2	119.000
-1.5	115.500
-1	106.000
-0.5	92.000
0	75.000
0.5	56.500
1	38.000
1.5	21.000
2	7.000
2.5	-2.500
3	-6.000
3.5	-2.000
4	11.000
4.5	34.500
5.0	70.000

9-12 (cont'd)

The method of nested multiplication is especially convenient when checking the values of a polynomial on either side of a value of x which may be a maximum, minimum or point of inflection. Thus, this technique is useful in calculus and senior math.

Order of Operations and Definition of Division

Rewrite the following so it can be evaluated on your calculator.

$$\cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1 - x}}}$$

Use various values for x.

Formula Substitution and Order of Operations

1. The formula $a = \dfrac{W}{\frac{W}{32.2} + \frac{I}{R^2}}$ is used to compute the acceleration of a wrecking ball.
   W = weight of ball
   I = inertia of cable wheel
   R = radius of cable wheel
   Rewrite the formula so it can be used on your calculator more conveniently.
2. $P = VI - I^2R$ is a formula for the power output of a direct-current electric motor.
   P = power output in watts
   V = line voltage
   I = armature current
   R = armature resistance. Rewrite the formula so it can be used on your calculator.
3. A formula for the maximum number of lights x in a Christmas tree is $x - 3HW$.
   H = tree height in feet
   W = width of base in feet
   The minimum number of lights (I) is ½ of maximum. What happens to x as H and W change?

Property of Numbers and Order of Operations

1. The formula $d = \pi w \frac{c}{s}$ can be used to find the distance in inches traveled by a bicycle.
   d = distance traveled in inches
   π = 3.14
   w = diameter of wheel in inches
   c = number of teeth on a chainwheel
   s = number of teeth on rear sprocket

(9-12 cont'd)

Use your calculator and fill in the following chart for (27 inch) (68.6 cm) bicycle.

c	s	d
50	12	in
	24	359.2 cm

2. Additional formulas that can be used with bicycles are:

a. $s = \frac{dr}{1056}$

s = speed in m.p.h.
r = revolutions per minute of pedals
d = distance traveled in inches

b. $s = \frac{3dr}{500}$

s = speed in kilometers
d = distance traveled in centimeters
r = number of pedal revolutions per minute

Exponents and Logarithms

1. A certain type of bacteria doubles every hour. At the end of 21 hours, one bacteria will multiply and completely cover a slide. How long will it take to cover one-half a slide? Use your calculator to help you. Fill in the following table.

Hour	Bact.
0	1
1	2
2	4
3	8

2. The formula $y = ne^{kt}$ is used to relate bacterial growth to time.
e = 2.72 (approximately)
k = constant depending on type of bacteria
t = time
n = starting number of bacteria
y = ending number of bacteria

How much time will it take 100 bacteria to increase to 600 bacteria when k = .872?

Ans: $t = \frac{\log 6}{8.872 \cdot \log 2.72}$

9-12 (cont'd)

3. The formula in 2 can be used to relate decay of radioactive elements to time. (y is final mass and n is original mass.) Example:
t = 20 years,
n = 250 milligrams,
y = 50 milligrams. Find k.

4. The formula $p - ie^{-kh}$ (where i is original intensity of energy, p is intensity of the energy passing through a medium, h is thickness of medium and k is a constant) is used to relate radient energy before and after it passes through a medium. Use your calculator to find out how p changes as h changes.

Nth Root of a Number

1. Estimate the following square root on your calculator (a four function calculator)

$\sqrt{200}$ and use Newton method

a. Trial Divisor 1214.
b. Divide 200 by 12. $200 \div 14 = 14.28$
c. Average 14 and 14.28 for new division

2. The following formulas can be used for Nth root of numbers.

$$\sqrt{N} = \frac{\frac{N}{D} + D}{2}$$

$$\sqrt[3]{N} = \frac{\frac{N}{D^2} + 2D}{3} = \frac{N \div D \div D + (D + D)}{3}$$

$$\sqrt[a]{N} = \frac{\frac{N}{D^{a-1}} + D(a - 1)}{a}$$

9-12 (cont'd)

Law of Sines

Find b if A = 43°, B = 17° and a = 100

Ans: $b = 100 \frac{\sin 17^\circ}{\sin 43^\circ} = 42.87$

Law of Cosines

Find A if a = 40, b = 64, c = 36

Ans: $A = \arccos \frac{(b^2 - c^2 - a^2)}{2bc} = 34^\circ 40'$

Polar Graphs

Graph $r = \sin\theta - 2$

Ans: Use sine function key to calculate successive values of θ at 15° intervals.

Fun With Numbers: A Limiting Property of All Fractions

This activity leads to a surprising result--the limiting value is $\sqrt{2} = 1.4142...$

Carry out the following algorithm:

a. write any fraction $\frac{a}{b}$ example: $\frac{13}{5}$

b. make a second fraction whose denominator is the sum of a and b; and whose numerator is the sum of b and a + b. $\frac{a+b}{13+5}$ $\frac{b+a+b}{a+b}$ $\frac{5+13+5}{13+5}$

c. continue the algorithm on the resulting fraction several times. $\frac{13}{5}$ to $\frac{23}{8}$ to $\frac{59}{41}$ to $\frac{141}{100}$ to $\frac{341}{241}$

d. compute the decimal equivalent of each fraction. Does it approach $\sqrt{2}$? 2.6 to 1.2778 to 1.439 to 1.41 to 1.4149 to...

e. Continue with $\frac{341}{241}$ another four times.

f. Begin with $\frac{1}{10}$. Develop a sequence of 8 fractions. Display the decimal value for each one.

9-12 (cont'd)

Fun with Numbers:
H.A.M. Numbers

Consider the number 100 as the sum 100 = 12 + 20 + 4 + 64 and the numbers p = 4, q = 16. Notice that adding, subtracting, multiplying and dividing 12, 20, 4 and 64 by 4 gives 16 each time.

$$12 + 4 = 16$$
$$20 - 4 = 16$$
$$4 \times 4 = 16$$
$$64 \div 4 = 16$$

and, $12 + 20 + 4 + 64 = 100$

The number 100 is an H.A.M. number because it is divisible by a square and there are a,b,c,d,p,q such that:

a. $a+b+c+d = 100 = m \cdot n^2$
b. $a + p = q$
c. $b - p = q$
d. $c \times p = q$
e. $d \div p = q$

Substituting for a,b,c,d, in (1) you find $mn^2 = \frac{q}{p}(p+1)^2$ or, for n prime,

$n = p + 1$ and $q = pm$.

These equations allow you to construct H.A.M. numbers as follows:

To construct a H.A.M. number	Illustration
f. Select any integer, m.	m = 65
g. Take any prime no., n.	n = 7
h. Form, H.A.M. = mn^2	H.A.M = 3185
To determine p and q	
i. $p = n - 1$	p = 6
j. $q = mp$	q = 390
To find a,b,c, and d	
k. $a = q - p$	a = 384
l. $b = q + p$	b = 396
m. $c = q \div p$	c = 65
n. $d = q \times p$	d = 2340

Summary

H.A.M. = 3185
384 + 6 = 390
396 - 6 = 390
65 × 6 = 390
2340 ÷ 6 = 390
384 + 396 + 65 + 2340 = 3185

9-12 (cont'd)

Practice Exercises

1. Use the calculator in the following exercise to find H.A.M. a,b,c,d, p and q.

   a. Take m = 2, n = 13
   b. Take m = 9, n = 5
   c. Take m = 37, n = 57

(The above materials were developed from a number curiosity described by Helen A. Merrill in Mathematical Excursions published by Dover Publications in 1933.)

Fun with Numbers:
Day of the Week

The formula below can be used to determine the day of the week, given the month, day and year.

$D = [\ \{(1.25Y + d + c) \div 7 - Wp\} \times 7.1]$

D = day of week, i.e. Friday

Y = year, i.e. 1927

d = day of month, i.e. 17

c = correction for month

Wp = greatest integer in $(1.25Y + d + c) \div 7$

[ ] represents greatest integer function.

Table for c:

Month	J	F	M	A	M	J	J	A	S	O	N	D
c (not leap year) =	6	2	2	5	0	3	5	1	4	6	2	4
c (leap year) =	*5*											

Table for D:

Day	S	M	T	W	T	F	Sa
D =	1	2	3	4	5	6	0

Example: October 13, 1925
Y = 1925, d = 13, c = 6

$[\ \{(1925 \times 1.25 + 13 + 6) \div 7 - Wp\} \times 7.1] = D...$

$[\ \{(2406.25 + 13 + 6) \div 7 - Wp\} \times 7.1] = D...$

$[\ \{2425.25 \div 7 - Wp\} \times 7.1] = D...$

$[\ \{346.46428... - 346\} \times 7.1] = D...$

$[\ \{.46428...\} \times 7.1] = D...$

$[3.296428] = D...$

$3 = D...$

or, day is Tuesday!
Try your birthday!

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